

NAME _____
AP CALCULUS AB SUMMER ASSIGNMENT 2017

DIRECTIONS: Each part must be completed separately on looseleaf. All work should be shown and done in a neat and precise manner. Any questions pertaining to the examples you may email me chorihan@moorechs.org. These assignments will be graded and you will have a test within the first two weeks of school in September. You are responsible to memorize the formulas and definitions.

Part One: Solve algebraically and check. Be sure to show all work.

1. $2x - 11 = 5x + 1$

7. $\frac{1}{2}(6x - 12) = -18$

2. $e^{3x} - 1 = 0$

8. $25^{x-2} = 125^{x+2}$

3. $5(x + 2) = 6 + 2(2x - 1)$

9. $\frac{x-5}{x} = \frac{x+4}{x}$

10. 9

4. $4(x + 5) = 5x - 2(x + 11)$

10. $4x^{\frac{1}{2}} - 1 = 3$

5. $\frac{3x-2}{x+2} = 0$

11. If $2x^2 - yx = 4$ and $x = -1$ find the value of y .

6. $\log_3 4 + \log_3 (x + 2) = 3$

12. $2x^{-4} + 1 = 163$

Part Two: Factor each of the following completely:

1. $5x^2 - 500$

9. $x^2 - 121$

2. $2x^2 + 8x + 8$

10. $19x^3 - 19x$

3. $25x^2 - 10x + 1$

11. $169 - m^2$

4. $x^3 + 3x^2 - 10x$

12. $7a^4b^2 - 28a^2b^2$

5. $y^2 + 10y + 16$

13. $x^4 - 81$

6. $x^2 - 2x - 63$

14. $4x^2 - 8x + 4$

7. $x^2 - 7x + 10$

15. $x^3 - 7x^2 - 9x + 63$

8. $x^2 - x - 42$

16. $12x^2 - 48$

PART THREE: Solve for x by factoring. Show all work for full credit.

1. $x^2 - 7x = 2x + 10$

2. $x^3 - 5x^2 - 4x + 20 = 0$

3. $7x^2 - 13x = 2$

4. $x^4 = 81$

5. $5x^2 - x = 4$

6. $10x(x - 2) = x(x - 1) - 13x - 1$

7. $9x^2 - 5 = 0$

8. $\frac{1}{4}x^2 - x + 1 = 0$

9. $7x^2 - 16x + 4 = 0$

10. $x^4 - 10x^2 + 9 = 0$

PART FOUR: Show all work on looseleaf or no credit will be given.

1. Write the equation of the line perpendicular to $3y - x = 2$ and passing through $(9, -10)$

2. Write the equation of the line normal to $4y - 2x = 1$ and passing through $(-10, 11)$

3. Write the equation of the line passing through the points $(-2, 5)$ and $(-5, 15)$

4. Write the equation of the circle with diameter 25 and having the center $(2, 1)$ in two different forms.

5. Write the equation of the circle where the endpoints of the diameter are $(2, 1)$ and $(-6, 7)$

6. Find the **exact values** of the following:

a. $\sin 150^\circ$ b. $\cos 330^\circ$ c. $\tan \frac{5\pi}{4}$ d. $\csc(-480^\circ)$ 5. $\sec\left(\frac{7\pi}{6}\right)$

7. Find the x -intercepts and y -intercepts of the following in ordered pair form.
- a. $y = x^2 + 6x - 16$ b. $4x^2 + 5y^2 = 20$ c. $x = y^2 - 2y - 3$
8. Find in terms of π the volume of a sphere that has a diameter of 20 inches.
9. Find to the nearest tenth the surface area of a sphere that has a radius of $\frac{1}{4}$
10. Find to the nearest hundredth the volume of a right cylinder with a height of 4 inches and diameter of 5 inches.
11. If the area of the great circle of a sphere is 144π , find the volume and surface area of the sphere in terms of π .
12. Find to the nearest tenth the diameter of a sphere that has a volume of 566 cubic feet.

PART FIVE: Show all work on looseleaf.

1. If $f(x) = x^2 - 2x + 1$ and $g(x) = 5x - 5$ find the following and simplify completely.
- a. $f(g(x))$ b. $g(f(x))$ c. $f(x)/g(x); x \neq 1$ d. $g(x) - f(x)$
2. If $\sin x = \frac{4}{5}$ find the exact value of $\cos(2x)$ where x is a positive acute angle.
3. Find the exact value of $\sin(15^\circ)$.
4. Find the exact value of the $\cos(45^\circ)$
5. If the $\sin A = \frac{1}{2}$ and angle A lies in quadrant II, find the exact value of the following. Be sure to use the double angle formulas.
- a. $\cos(2A)$ b. $\tan(2A)$ c. $\sin(2A)$
6. Simplify completely the following:

a. $4\sin^2 A + 4\cos^2 A$

b. $\sqrt{x^2 - 6x + 9}$

7. Find the amplitude, frequency and period of $y = -3\sin x$

8. Sketch the graph of $y = 4\cos x$ from $-\pi \leq x \leq \pi$

PART SIX: Write each of the following in simplest radical form. For #s 5 - 12, perform the indicated operation and express your answers in simplest radical form.

1. $3\sqrt{20} - \sqrt{45}$

6. $\sqrt{300} - 4\sqrt{27} + \sqrt{12}$

2. $2\sqrt{90} + 5\sqrt{40}$

7. $(5 - 2\sqrt{2})(7 + \sqrt{2})$

3. $(3\sqrt{7})^2$

8. $(3 + 2\sqrt{11})^2$

4. $(5 - \sqrt{2})^2$

9. $11\sqrt{45} - 2\sqrt{20}$

5. $(5\sqrt{x})^2$

10. $(3\sqrt{x})^3$

PART SEVEN: Using the given functions, evaluate 1 – 7. Show all work or no credit will be given.

$f(x) = 6 - 3x$ $g(x) = \frac{1}{4}x - 11$ $h(x) = x^2 - x - 1$

1. $f(10)$

2. $f(-3)$

3. $g(-8)$

4. $g(200)$
5. $h(-4)$
6. $h(-2)$
7. $f(1) + g(3)$

DEFINITIONS

The following is a list of terms and properties which are necessary for success in Calculus. You will be tested on these terms during your first week of classes.

abscissa - The horizontal or x -coordinate of a two-dimensional coordinate system.

absolute value - The distance from 0 to a number n on a number line. The absolute value of a number n is indicated by **| n |**

algebraic equation - A mathematical statement that is written using one or more variables and constants which contains an equal sign.

algebraic expression - A mathematical phrase that is written using one or more variables and constants, but which does not contain the symbols $=$, $<$, $>$, \leq , \geq , or \neq .

base of an exponential function - The number b in the exponential function $y = b^x$.

center-radius equation of a circle - The form of the equation of a circle with center (h, k) and radius r given by the formula $(x - h)^2 + (y - k)^2 = r^2$

Example: If the coordinates of the center of the circle are $(3, -8)$ and the length of the radius is 6, then the equation of the circle is $(x - 3)^2 + (y + 8)^2 = 36$.

circle - The set of all points (or locus of points) in a plane that are a fixed distance, (called the radius) from a fixed point, (called the center).

coefficient - The numerical factor of a term in a polynomial.

Example: 9 is the coefficient of $9x^5$

common factor - A number, polynomial, or quantity that divides two or more numbers or algebraic expressions evenly.

conjugate - Two binomials whose first terms are equal and last terms are opposites.

cosine - For a given acute angle in a right triangle, the ratio of the length of the side adjacent to an acute angle to the length of the hypotenuse.

degree of a monomial - The sum of the exponents of the variables in the monomial.

Example: The degree of the monomial $3x^5$ is five.

The degree of the monomial x^3y^4 is seven.

The degree of xy is two.

The degree of 7 is zero.

degree of a polynomial - The highest degree of any monomial term in the polynomial.

difference of two perfect squares - A binomial of the form $a^2 - b^2$ which can be factored into $(a - b)(a + b)$.

direct variation - A relationship in which the ratio of two variables is constant. A direct variation has an equation in the form $y = kx$ where x and y are variables and k is the constant of variation.

domain - The set of values of the independent variable, usually x in an equation, for which a given function is defined; the set of first coordinates in the ordered pairs of a relation.

equation - A mathematical sentence stating that two expressions are equal.

exact answer - The solution to an equation that has not been rounded.

exact value - The value of an expression that has not been rounded.

exponential function - A function with a variable in the exponent; an equation in the form $y = ab^x$, where $a \neq 0$, $b > 0$ and $b \neq 1$.

extraneous root/value - A solution of a derived equation that is not a solution of the original equation.

factor - (noun) A whole number that is a divisor of another number; an algebraic expression that is a divisor of another algebraic expression.

Example: 3 is a factor of 12

factor - (verb) Find the number of algebraic expressions that give an indicated product.

Example: To factor $x^2 + x - 6$, write $(x + 3)(x - 2)$.

function - A rule that assigns to each number x in the function's domain a unique number y in the range.

geometry - Branch of mathematics that deals with the properties, measurement, and relationships of points, lines, angles, surfaces, and solids.

greatest common factor (GCF) - The greatest number or expression that is a factor of two or more numbers or expressions.

index of a radical - The letter n in the expression $\sqrt[n]{\quad}$

irrational number - A number that cannot be expressed as the ratio of two integers, that is, in $\frac{a}{b}$ form.

laws of exponents - Rules involving operations on expressions with like bases.

Example: Multiplication law: $x^4x^2 = x^6$

Division law: $\frac{x^8}{x^3} = x^5$

Power law: $(x^5)^3 = x^{15}$

linear equation - A first degree equation.

Examples: $y = 6x + 8$

$$12n - 19 = 23 + 7n$$

linear system - A set of two or more linear equations with common variables.

measure of central tendency - A summary statistic that indicates the typical value or center of an

negative exponent - An exponent that is a negative number. In general, $x^{-a} = \frac{1}{x^a}$, $x \neq 0$

opposite side in a right triangle - The side across from an angle.

ordinate - The vertical coordinate of a two-dimensional rectangular coordinate system. It is the y -value in a set of ordered pairs (x, y) .

parabola - The graph of a quadratic function. It is two-dimensional and u-shaped. The equation of a parabola is in the form $ax^2 + bx + c = 0$.

pi - The irrational number equal to the length of the circumference of a circle divided by the length of its diameter. The symbol for pi is π .

principal square root - The positive square root of a number.

quadratic equation - An equation that can be written in the form $ax^2 + bx + c = 0$, where a , b , and c are real constants and $a \neq 0$.

quadratic formula - The formula used to determine the roots of the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

radical - The root of a quantity as indicated by the radical sign.

radicand - The quantity under a radical sign; a number or expression from which a root is extracted.

rational number - Any number that can be expressed as a ratio in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. A rational number is either a terminating or repeating decimal.

root of an equation - A solution to an equation when $y = 0$.

Example: A root of the equation $y = 6x - 18$ is 3 because when 3 is substituted in for x , the

sine - For a given acute angle in a right triangle, sin is the ratio of the length of the side opposite the acute angle to the length of the hypotenuse.

slope - The measure of the steepness of a line; the ratio of vertical change to horizontal change; if point P is (x_1, y_1) and point Q is (x_2, y_2) the slope of $PQ = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

solution set - Any and all value(s) of the variable(s) that satisfy an equation, inequality, system of equations, or system of inequalities.

tangent (of an angle) - For a given acute angle in a right triangle, tan is the ratio of the length of the side opposite the acute angle to the length of the side adjacent to the angle.

trigonometric functions - The functions, sine, cosine, tangent, cotangent, secant and cosecant.

undefined - An expression in mathematics which does not have meaning and therefore is not assigned a value. When the denominator of a fraction is equal to 0, the fraction is undefined.

Example: When $x=4$, the expression $\frac{x+9}{x-4}$ is undefined.

univariate - A set of data involving one variable.

variable - A quantity whose value can change or vary; in algebra, letters often represent variables.

Properties of Mathematics

Commutative Property of Addition
 $a + b = b + a$

Commutative Property of Multiplication
 $ab = ba$

Associative Property of Addition
 $a + (b + c) = (a + b) + c$

Associative Property of Multiplication
 $a \times (b \times c) = (a \times b) \times c$

Identity Property of Addition
 $a + 0 = a$

Identity Property of Multiplication
 $a \times 1 = a$

Additive Inverse Property
 $a + -a = 0$

Multiplicative Inverse Property
 $a \times \frac{1}{a} = 1$

Distributive Property
 $a(b + c) = ab + ac$
 $a(b - c) = ab - ac$

Multiplication Property of Zero
 $a \times 0 = 0$

TRIGONOMETRIC IDENTITIES

You are required to know the following identities:

1. $\cos(2A) = 2\cos^2A - 1 = 1 - 2\sin^2A = \cos^2A - \sin^2A$
2. $\sin(2A) = 2\sin A \cos A$
3. $\sin^2A + \cos^2A = 1$
4. $1 + \tan^2A = \sec^2A$

5. $1 + \cot^2 A = \csc^2 A$

6. $\tan A = \sin A / \cos A$

7. $\cot A = \cos A / \sin A$

The following reciprocal functions:

1. $\csc A = 1 / \sin A$

2. $\sec A = 1 / \cos A$

3. $\cot A = 1 / \tan A$

The following exact values:

| | 30° | 45° | 60° | |
|-----|-----------------------|----------------------|-----------------------|--|
| sin | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | |
| cos | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | |
| tan | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | |
| csc | 2 | $\sqrt{2}$ | $2\frac{\sqrt{3}}{3}$ | |
| sec | $2\frac{\sqrt{3}}{3}$ | $\sqrt{2}$ | 2 | |
| cot | $\sqrt{3}$ | 1 | $\frac{\sqrt{3}}{3}$ | |
| | | | | |